A. Appendix

A.1. Integration limits

In this section, we show how the limits for the integrations along the rays of a constant impact parameter $p = r\ (1-\mu^2)^{1/2}$ can be modified to account for the flared disk geometry.

Let us consider a midplane point P or a boundary point U lying at the radial distance r from the central star in the shaded two-sector area shown in Fig. 2. To find the radiation moments (19), (20), (21) at one of the points, it is necessary to integrate the radiative transfer equations (2) and (3) over all rays passing through that point, for only those parts of the rays which lie inside the cross-sectional disk area which we describe by the angle $\alpha = \angle\, BOC$. Each ray may have in general from 2 to 4 intersection points with the disk boundaries. We have to repeat the ray tracing for a set of such cross-sections of different inclinations ϕ to the midplane, from $\psi/2$ to $\pi/2$. An integration over all ϕ completes the radiation moments at the given midplane or boundary point.

A.1.1. Midplane points

For a point P in the disk midplane (Fig. 2), α is related to the opening angle ψ and to the midplane azimuthal angle $\phi = \angle\, EBC + \psi/2$ between the cross-section and the midplane by

$$\alpha = \arcsin\left[\frac{\sin(\psi/2)}{\sin\phi}\right]. \tag{A1}$$

The angle α decreases from $\pi/2$ to $\psi/2$ for $\phi \in [\psi/2, \pi/2]$. A symmetry of the point P position in the cross-section implies that we need to integrate over μ only from 0 to 1, as in spherical geometry. Therefore, we have to find five intersection points with the lines GH, OH, OCB, OA, and AB (Fig. 2); one can derive the corresponding radii

$$r_{GH} = R_2, \qquad \theta \le \arctan\xi\,, \tag{A2}$$

$$r_{OH} = \frac{p}{\sin(\alpha-\theta)}, \qquad \theta < \arctan\xi\,, \tag{A3}$$

$$r_{OCB} = \min\left[\frac{p}{\sin(\alpha+\theta)}, R_2\right], \tag{A4}$$

$$r_{OA} = \frac{p}{\sin(\theta-\alpha)}, \qquad \theta > \arctan\zeta\,, \tag{A5}$$

$$r_{AB} = R_2, \qquad \theta \le \arctan\zeta\,, \tag{A6}$$

where we have defined

$$\xi = \frac{R_2\sin\alpha}{r + R_2\cos\alpha}\,, \qquad \zeta = \frac{R_2\sin\alpha}{-r + R_2\cos\alpha}\,. \tag{A7}$$

For a given impact parameter p, one should integrate I_ν^+ along the ray, from r_{GH} to r_{OH} and from r_{OCB} to the radial position r of the point P. The inward directed intensity I_ν^- should be integrated from r_{AB} or r_{OA} (depending on the ray direction θ) to r.

A.1.2. Boundary points

For a point U in the disk's upper (or lower) boundary (Fig. 2), α is defined by Eq. (A1), while the boundary azimuthal angle $\phi_b = \angle\, BCT$ is related to ψ and ϕ by

$$\phi_b = \arccos\left[\frac{\cos\phi}{\cos(\psi/2)}\right]. \tag{A8}$$

The angle ϕ_b increases from 0 to $\pi/2$ for $\phi \in [\psi/2, \pi/2]$. For the point U position in the cross-section, there is no symmetry of the latter about the $\mu = 0$ direction. Therefore, we have to integrate the intensities over μ from -1 to 0 and from 0 to 1, in each interval separately.

(1). For $0 < \mu < 1$, we have to find five intersection points with the lines FH, OH, OC, OA, and AC (Fig. 2); one can derive the corresponding radii

$$r_{FH} = R_2, \qquad \theta \le \arctan\xi_1\,, \tag{A9}$$

$$r_{OH} = \frac{p}{\sin(2\alpha-\theta)}, \qquad \theta < \arctan\xi_1\,, \tag{A10}$$

$$r_{OC} = r\,, \tag{A11}$$

$$r_{OA} = \frac{p}{\sin(\theta-2\alpha)}, \qquad \theta > \arctan\zeta_1\,, \tag{A12}$$

$$r_{AC} = R_2, \qquad \theta \le \arctan\zeta_1\,, \tag{A13}$$

where we have defined

$$\xi_1 = \frac{R_2\sin(2\alpha)}{r + R_2\cos(2\alpha)}\,, \qquad \zeta_1 = \frac{R_2\sin(2\alpha)}{-r + R_2\cos(2\alpha)}\,. \tag{A14}$$

For a given impact parameter p, one should integrate I_ν^+ along the ray, from r_{FH} to r_{OH}. The inward directed intensity I_ν^- should be integrated from the radial position r_{OC} of the point U to r_{OA} or r_{AC} (depending on θ).

(2). For $-1 < \mu < 0$, we have to find five intersection points with the lines AC, OA, OC, OH, and FH (Fig. 2); the latter two intersections exist only for the cross-sections with $2\alpha > \pi/2$. The corresponding radii are

$$r_{AC} = R_2, \qquad \theta \ge \arctan\xi_2\,, \tag{A15}$$

$$r_{OA} = \frac{p}{\sin(\theta-2\alpha)}, \qquad \theta < \arctan\xi_2\,, \tag{A16}$$

$$r_{OC} = r\,, \tag{A17}$$

$$r_{OH} = \frac{p}{\sin(2\alpha-\theta)}, \qquad \theta > \arctan\zeta_2\,, \tag{A18}$$

$$r_{FH} = R_2, \qquad \theta \le \arctan\zeta_2\,, \tag{A19}$$

where we have defined

$$\xi_2 = \frac{R_2\sin(2\alpha)}{r - R_2\cos(2\alpha)}\,, \qquad \zeta_2 = \frac{R_2\sin(2\alpha)}{-r - R_2\cos(2\alpha)}\,. \tag{A20}$$

For a given impact parameter p, one should integrate I_ν^+ along the ray, from r_{AC} or r_{OA} (depending on θ) to the radial position r_{OC} of the point U. The inward directed intensity I_ν^- should be integrated from r_{FH} to r_{OH}.

A.1.3. Emergent intensity and flux

To calculate the model emergent spectrum, we have to specify the direction of the line of sight which we describe by the viewing angle θ_v relative to the midplane of the disk; it may have any value between 0 and $\pi/2$. We define a set of the disk cross-sections by different planes containing the line of sight and passing through the stellar center (for an example of the disk cross-section, see Fig. 2). A separate ray tracing in the cross-section areas along this direction gives us the emergent intensity distribution; integrating the latter over the disk projection (over the azimuthal angle ϕ_v) we get the flux.

The angle ϕ_v is defined using the angle ϕ between the cross-section and the disk midplane as

$$\phi_v = \arccos\left[\frac{\cos\phi}{\cos\theta_v}\right]. \tag{A21}$$

The angle α describing the cross-section geometry is given by Eq. (A1). An expression for the angle ω between the viewing direction and the line GB in the disk midplane (Fig. 2) depends on whether θ_v is smaller or larger than $\psi/2$.

(1). For $\theta_v \leq \psi/2$, $\phi \in [\psi/2, \pi/2]$ and one can see that $\alpha \in [\pi/2, \psi/2]$; in this case, ω is defined by the expression

$$\omega = \arcsin\left[\frac{\sin\theta_v}{\sin\phi}\right]. \tag{A22}$$

(2). For $\theta_v > \psi/2$, $\phi \in [\theta_v, \pi/2]$ and one can easily verify that $\alpha \in [\alpha_{max}, \psi/2]$; the maximum angle α_{max} of the cross-sectional area is defined by Eq. (A1) with $\phi = \theta_v$. In this case, ω can be computed from the formula

$$\omega = \arccos\left[\cos\theta_v \frac{\sin\phi_v}{\sin\phi}\right]. \tag{A23}$$

Defining appropriate values of the ray direction θ for the mid-plane or boundary points (as a function of α and ω), it is possible to use the same expressions for the integration limits (Sects. A.1.1, A.1.2) to calculate emergent intensity and flux. Of course, it is necessary to integrate I_ν^+ over the whole body of the disk, from the "background" boundaries to those seen by the observer.

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